Note On Analytic Functors As Fourier Transforms.

Brian J. DAY

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Abstract: *Several notions of "analytic" functor introduced recently in the literature fit into the graphic fourier transform context presented in [D].*

Various concepts of "analytic" functor are well characterized in different places in the literature. Here we want to mention explicitly how two ideas, introduced in [AV] and [FGHW], can also be viewed from [D]. However, we don't offer further characterization results here.

Example [FGHW]:

Let $N : \mathscr{BA} \to [\mathscr{A}^{\mathrm{op}}, \mathbf{Set}]$ be the canonical "inclusion" functor from the (monoidal) groupoid $\mathscr{BA}$ constructed in [FGHW] from the small $\mathbf{Set}$-category $\mathscr{A}$; i.e. $\mathscr{BA}$ is the groupoid of all isomorphisms in the free finite-coproduct completion of $\mathscr{A}$ in $[\mathscr{A}^{\mathrm{op}}, \mathbf{Set}]$. Then, assuming (here) that $\mathscr{C}$ is small, the functor

$$\exists_N : [\mathscr{BA}, [\mathscr{C}^{\mathrm{op}}, \mathbf{Set}]] \to [[\mathscr{A}^{\mathrm{op}}, \mathbf{Set}], [\mathscr{C}^{\mathrm{op}}, \mathbf{Set}]]$$

which is precisely the process of left Kan extension along N, is conservative (because $\mathscr{BA}$ is a groupoid) and tensor product preserving (because N preserves finite coproducts). This is then consistent with [D] for $\mathscr{V} = [\mathscr{C}^{\mathrm{op}}, \mathbf{Set}]$ (cartesian monoidal).

Example [AV]:

In [AV] Remark 4.5, the authors complain (justly) that many of their "analytic" functors for $\mathscr{V} = \mathbf{Vect}_k$ are not k-linear. But this is not too serious a matter because the set-up in [D] permits a k-linearization (this is not a tautology, but merely an adjunction). Thus the "ordinary" kernel discussed in [AV] §4, namely

$$K : \mathscr{B} \times \mathscr{V}_0 \longrightarrow \mathscr{V}_0, \ K(n, X) = \otimes^n X,$$

yields the corresponding (multiplicative) $\mathscr{V}$-kernel

$$E : k_* \mathscr{B} \otimes k_* \mathscr{V}_0 \xrightarrow{\ \cong\ } k_*(\mathscr{B} \times \mathscr{V}_0) \xrightarrow{\ k_* K\ } k_* \mathscr{V}_0 \xrightarrow{\ can.\ } \mathscr{V}$$

where k_* denotes the free-$\mathbf{Vect}_k$-structure functor, and $\mathscr{V}_0$ is the ordinary category underlying $\mathscr{V}$ (here $k_* \mathscr{V}_0$ has the comonoidal structure directly induced by that on $\mathscr{V}_0$). Then the $\mathscr{V}$-functor

$$\overline{E} : [k_* \mathscr{B}, \mathscr{V}] \longrightarrow [k_* \mathscr{V}_0, \mathscr{V}],$$

is conservative (since $\mathscr{B}$ is a groupoid) and tensor product preserving (since E is multiplicative). The Fourier transforms $\overline{E}(f)$ can thus be viewed as either k-linear "E-analytic" functors

$$k_*\mathscr{V}_0 \longrightarrow \mathscr{V},$$

or just ordinary "[AV]-analytic" functors

$$\mathscr{V}_0 \longrightarrow \mathscr{V}_0,$$

in the sense of [AV] Definition 4.1. Then the considerations of [D] Section 1.3 apply.

Remark 1 *The term "analytic" functor seems quite appropriate in such cases.*

Example [D]:

A type of "quantum category" example evolves from any $\mathscr{V}$-promonoidal category $(\mathscr{A}, p, j)$. Namely, the left "Cayley" functor

$$\overline{K} : [\mathscr{A}, \mathscr{V}] \longrightarrow [\mathscr{A}^{\mathrm{op}} \otimes \mathscr{A}, \mathscr{V}]$$

given by

$$\overline{K}(f)(A, B) = \int^{X} p(X, A, B) \otimes f(X),$$

the $\mathscr{V}$-kernel functor

$$K : \mathscr{A}^{\mathrm{op}} \otimes \mathscr{A}^{\mathrm{op}} \otimes \mathscr{A} \longrightarrow \mathscr{V}$$

here being just the promultiplication p. This $\overline{K}$ is both conservative and tensor preserving, where $[\mathscr{A}, \mathscr{V}]$ has the convolution structure and $[\mathscr{A}^{\mathrm{op}} \otimes \mathscr{A}, \mathscr{V}]$ has the tensor product defined by bimodule composition. Thus $\overline{K}$ qualifies as a "Fourier transformation" [D].

References.

[AV]   J. Adamek and J. Velebil, "Analytic functors and weak pullbacks",
          Theory Appl. Categories, 21(11), (2008) 191-209.

[D]   B. J. Day, "Monoidal functor categories and graphic Fourier transforms",
          arXiv:mathQA/0612496v1, 18 Dec. 2006.

[FGHW]   M. Fiore, N. Gambino, M. Hyland and G. Winskel, "The cartesian closed
          category of generalized species of structures", London Math. Soc. (2007), 1-18.

Mathematics Dept., Faculty of Science, Macquarie University, NSW 2109, Australia.

Any replies are welcome through Tom Booker (thomas.booker@students.mq.edu.au),
who kindly typed the manuscript.